Name: Key

INSTRUCTIONS

1. Nonprogrammable calculators (or a programmable calculator cleared in front of the professor before class) are allowed. Exam is closed book/closed notes.
2. There are 4 pages front and back for the exam in addition to the cover sheet and the table page. You may tear off the last page (tables and equations) to make your work easier. However, this sheet must be returned.
3. Please use at least 3 decimal places in all answers that are not exact unless mentioned otherwise in the question.
4. Work is required to receive credit. Partial credit will be given for work that is partially correct. Points will be deducted for incorrect work even if the final answer is correct.
5. Please attach all scrap paper and the equations page to the exam.
6. Good Luck!

Question	Possible	Score
3	17	
4	25	
5	30	
6	34	
Total	106	

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(5 pts.) 1. Write a question in English in which the mathematical problem is given below (NO calculations are required). There should be no symbols except possibility for an X in the question.

Given that X ~ Binomial(10,0.5). Find P(X = 2).

The key aspects are that you have 10 independent trials with two possibilities for each.

Toss a fair coin 10 times. What is the probability that there are 2 heads?

Roll a fair die (any number of sides) 10 times. What is the probability that 2 of the rolls are even?

If the situation does not imply independence, that it had to be stated explicitly in the question.

(12 pts.) 2. A large insurance agency services a number of customers who have purchased both a homeowner's policy and an automobile policy from the agency. For each type of policy, a deductible amount must be specified. For an automobile policy (X), the choices are \$100 and \$250 and \$500, whereas for a homeowner's policy (Y), the choices are \$0, \$100, and \$200. The joint mass is shown below. Some of the work for parts a) and b) may be shown directly on the table.

homeowner's policy, Y

automobile policy, X		0	100	200	$p_X(x)$
	100	0.20	0.10	0.15	
	250	0.05	0.15	0.15	
	500	0.00	0.10	0.10	
	$p_Y(y)$				

a) What is F(250,200)?

F(250,100) = p(100,0) + p(100,100) + p(100,200) + p(250, 0) + p(250,100) + p(250,200)
= 0.20 + 0.10 + 0.15 + 0.05 + 0.15 + 0.15 = 0.80

b) Are X and Y independent? Why or why not? Hint: You do not need to fill in all of the marginal distributions to explain your answer.

In this part, you had to show your work on how you obtained the marginal distributions to receive full credit.

You may use any of the cells in the table.

X and Y are independent if p(x,y) = p(x) p(y)
For x = 100, y = 0, p(x,y) = 0.20
$p_X(100) = 0.20 + 0.10 + 0.15 = 0.45$ $p_y(0) = 0.20 + 0.05 + 0.00 = 0.25$

$0.20 \neq (0.45)(0.25) = 0.1125$

(25 pts.) 3. Given the following mass,

x	-1	0	1	3
$p_X(x)$	0.2	?	0.3	0.1

a) What is the missing value?

P(0) = 1 – p(-1) – p(1) – p(3) = 1 – 0.2 – 0.3 – 0.1 = 0.4

b) What is the CDF for this situation? **Please show your work.**

$$F_X(x) = \begin{cases} 0 & 0 & x < -1 \\ 0 + 0.2 & 0.2 & 1 \le x < 0 \\ 0.2 + 0.4 & 0.6 & 0 \le x < 1 \\ 0.6 + 0.3 & 0.9 & 1 \le x < 3 \\ 0.9 + 0.1 & 1 & 3 \le x \end{cases}$$

Work is required as well as defining the cdf as a piecewise function.

c) What is the expected value of X?

$$\mathbb{E}(X) = \sum xp(x) = (-1)(0.2) + (0)(0.4) + (1)(0.3) + (3)(0.1) = -0.2 + 0 + 0.3 + 0.3 = 0.4$$

d) What is the standard deviation of X?

$$\mathbb{E}(X^2) = \sum x^2 p(x) = (-1)^2(0.2) + (0)^2(0.4) + (1)^2(0.3) + (3)^2(0.1) = 0.2 + 0 + 0.3 + 0.9 = 1.4$$

Var(X) = $\mathbb{E}(X^2)$ - $(\mathbb{E}(X))^2$ = 1.4 – 0.4^2 = 1.24

$$\sigma_X = \sqrt{Var(X)} = \sqrt{1.24} = 1.114$$

(18 pts.) 4. Now that you have found your Obi Wan Kenobi Watch (from the homework), you now want to find a Yoda watch. However, you only need one more watch so you will stop looking for watches once you have found one of them. Suppose that in one type of cereal, A, there are Yoda watches in 10% of the boxes. We want to know how many boxes of cereal you need to look in to find the Yoda watch.

a) What is the distribution including the parameters? Why did you choose this distribution?

Geometric. $p = 0.1$

It is a geometric distribution because it is binary, independent trials, constant P(S) and we want to know the number of trials until the first success.

b) What is the probability that you have to look in more than 15 boxes to find the Yoda Watch?

$P(X > 15) = q^{15} = (0.9)^{15} = 0.206$

c) In a second type of cereal, B, the Yoda watches are only included in 2% of the boxes. The Yoda watches in the two types of cereal are independent of each other. What is Var(2A – 3B)?

From the equation sheet:

$$Var(X) = \frac{q}{p^2}$$

$$Var(A) = \frac{0.9}{0.1^2} = 90 \qquad Var(B) = \frac{0.98}{0.02^2} = 2450$$

Var(2A – 3B) = 4Var(A) + 9 Var(B) = (4)(90) + (9)(2450) = 22,410

(12 pts.) 5. On spring break, you downloaded ten new movies to watch on your tablet. After you take this midterm, you are going to take a break and watch three of them. In this problem, to obtain full credit, you need to explicitly use factorials in your answers.

As mentioned on the board, this is unordered.

a) How many different ways can you watch the movies if you do not want to watch a movie more than once?

unordered, without replacement ==> combination

$$\binom{10}{3} = \frac{10!}{3!\,7!} = 120$$

b) How many different ways can you watch the movies if you don't mind watching the movies more than once?

unordered, with replacement ==> SB
stars = 3 (the number of movies that you watch), bars = 10 (all the movies)
A bin can have 0 in it.

$$\binom{3+10-1}{3} = \binom{12}{3} = \frac{12!}{3!\,9!} = 220$$

(12 pts.) 6. In a certain region, there are 25 high schools and 35 middle schools. You and five of your friends (six total) are randomly assigned student teaching at these schools. Because of the size of the schools, there can only be one math student teacher at each school. We are interested in the number of student teachers assigned to the high schools.

a) What is the distribution including the parameters? Why did you choose this distribution?

Hypergeometric. M = 25, N = 25 + 35 = 60, n = 6
This is a hypergeometric because this is binary with a student can only teach at one school so it is without replacement.

b) What is the probability that exactly four of you are assigned to the high schools?

$$P(X=4)=\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}=\frac{\binom{25}{4}\binom{35}{2}}{\binom{60}{6}}=0.1503$$

(13 pts.) 7. Suppose that the average number of tornadoes that hit a certain county is 1.5 per year. We are interested in the number of tornadoes that hit the county in a certain time period.

a) What is the distribution for the number of tornadoes including the parameters? Why did you choose this distribution?

Poisson, $\lambda = 1.5$
This is a Poisson because it is a rate (something per time)

b) What is the probability that at least 2 tornadoes will hit the county in the next 2 years.?

Note: $P(X \geq x) \neq P(X = x)$

$\lambda' = 2(\lambda) = 2(1.5) = 3$

$$P(X\geq 2)=1-P(X<2)=1-P(X=0)-P(X=1)=1-\frac{e^{-3}3^0}{0!}-\frac{e^{-3}3^1}{1!}=0.801$$

(9 pts.) 8. In 7-card draw poker, what is the probability that the hand contains 3 of one kind and 2 pair ({x,x,x,y,y,z,z}, the order of the cards is not important)? No factorials are required. (Please write down 6 decimal places in your answer.)

$$\frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{2}\binom{4}{2}}{\binom{52}{7}}=0.0009235$$

Numerator:
Term 1: number of ways you can choose the first number
Term 2: number of ways that you can choose 3 out of 4 cards
Term 3: number of ways that you can choose the second two numbers (they are indistinguishable so you can not do 12 x 11)
Term 4 and 5: number of ways that you can choose 2 out of 4 cards
Denominator: the number of ways that you can choose 7 cards out of the whole deck